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**ENGINEERING ANALYSIS**

**SUMMER 2015**

**Homework #3 Problem 3**

Consider the initial boundary value problem for the two-dimensional wave equation:

We will make the following substitutions into the differential equation, using the separation of the variables: ,

This yields the following equation,

Factor out the ‘h’ term on the right side to get,

Divide out both sides by common factors to get,

Therefore, we obtain the following differential equations:

Let us examine the second differential equation:

Now we can move terms to opposite sides of the equal sign and divide out common terms:

Now from the ψ & ϕ differential equation we obtain two more separate differential equations:

Let us analyze the first differential equation that involves ψ, the characteristic equation is:

So the solution to the ψ equation is of the form: . The boundary conditions have the following implication: . As from problem #2 we obtain a similar result for our function ψ: ; therefore, . From the second part of the boundary condition we get: ; therefore,

And the corresponding function is:

Now let us consider the differential equation involving ϕ:

The characteristic equation is,

Again we will have solutions in the form, . The boundary conditions have the following implication: . When x = 0 we get, ; therefore, . If we consider the second boundary condition when x = L we get, . Hence,

But previously we found μ, if we substitute we get,

The corresponding functions we be the following,

Therefore,

Thus we have established the following:

Let us now examine the temporal equation: . It has the following characteristic equation,

So our solution to the temporal equation will be of the form, recall we have found ,

Therefore the solution to our wave equation will be of the form,

If we let m = 1,2,3 then by superposition our solutions we be of the form,

Since each of these are solutions to the wave equation, then their sum is also a solution: , therefore,

Let us now use the first initial condition to get,

We will denote the inner summation as :

Notice that is a Fourier Sine series of f(x,y) and we can determine the coefficient by the following:

Recall, that we denoted the following,

Set both of these equalities equal to each other to get,

The expression on the right is a function on ‘x’ and the left is the Fourier Sine series of it; therefore, we can find an expression for the coefficients by,

If we rearrange terms we get,

We will now use the second initial condition to find the ‘B’ coefficients: . First let us differentiate the function u(x,y,t) with respect to ‘t.’

If we substitute t = 0 we get,

We will follow the same steps as we did for the first initial condition,

Therefore,

But here,

Equate the two expressions for and we obtain,

Therefore,

If we rearrange the terms we get,

We have established the solution to the wave equation with the given B.C. and I.C.: